Sum Rules for the Decay of the Baryon Decuplet into the Baryon and Meson Octets in Broken SU(3) Symmetry

VIRENDRA GUPTA* AND VIRENDRA SINGH†
Institute for Advanced Study, Princeton, New Jersey
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We have investigated the sum rules for the decuplet → octet+octet coupling constants, taking into account the SU(3)-symmetry-breaking interactions to first order. There are seven sum rules for the possible twelve coupling constants. One of these sum rules involves only the coupling constants for the decays $N^*\to N\pi$, $Y_1^*\to\Lambda\pi$, $Y_1^*\to\Sigma\pi$, and $\Xi^*\to\Xi\pi$, all of which are experimentally known. We also give the corresponding sum rules for $10^*\to 8+8$ decay.

I. INTRODUCTION

THE SU (3) octet model symmetry scheme[1,2] for strong interactions has had considerable success in classifying the various observed meson and baryon states into SU(3) multiplets. It successfully predicted the existence of new particles, the most impressive being the Ω^- baryon[3] required to complete the baryon decuplet. Apart from such qualitative results, there are a number of results obtained by taking into account the supposedly "small breaking" of SU(3) symmetry by a perturbation calculation. The Gell-Mann[1] and Okubo[4] mass formula or sum rule is a result of this type and illustrates the striking success of a first-order perturbation calculation. The consequences, of lowest order breaking of SU(3), have been investigated for

(i) electromagnetic form factors,[5]
(ii) electromagnetic mass splittings in SU(3) multiplets,[5]
(iii) sum rules for octet-octet-octet coupling constants.[6]

Of these, (i) and (iii) cannot be checked at present though calculation (ii) agrees rather well with experiment. Thus, it is desirable to search for results similar to above which are amenable to experimental verification and so furnish more quantitative evidence for or against SU(3) symmetry.

In this note[7] we work out the consequences of a broken SU(3) symmetry for the decays of the $J^P=\frac{3}{2}^+$ baryon decuplet D into the $J^P=\frac{1}{2}^+$ baryon octet B and the $J^P=0^-$ meson octet P.[7] We find seven sum rules between the twelve $[DBP]$—coupling constants. One of these, fortunately, involves only the coupling constants for the observed decays $N^*\to N\pi$, $Y_1^*\to\Lambda\pi$, $Y_1^*\to\Sigma\pi$, and $\Xi^*\to\Xi\pi$ and can thus be checked experimentally. We now proceed to give a derivation of our main results (Sec. III). However, before doing this we illustrate our use of the "spurion method" with a simpler example in Sec. II, and then use it for SU(3). The use of the "spurion method" simplifies the work enormously. Finally, we discuss the sum rules in Sec. IV.

II. ISOSPIN VIOLATION BY ELECTROMAGNETISM

We will consider the decay of the resonance N^*(1238 MeV) with isospin $I=\frac{3}{2}$ and $J=\frac{3}{2}^+$ into nucleon N and pion π. For exact isospin symmetry there is one coupling constant f_0 for the decays $N^*\to N+\pi$, in terms of which the six coupling constants $F(N_3^*,p\pi^+)$, etc., are given as follows:

$$\begin{aligned}\sqrt{2}F(N_3^*,p\pi^+)&=\sqrt{3}F(N_1^*,p\pi^0)\\&=6^{1/2}F(N_1^*,n\pi^+)=6^{1/2}F(N_{-1}^*,p\pi^-)\\&=\sqrt{3}F(N_{-1}^*,n\pi^0)=\sqrt{2}F(N_{-3}^*,n\pi^-)=f_0\,,\end{aligned}$$

where we have used $N_{2I_3}^*$ to denote a component of the N^* multiplet.

We assume that isospin violating interactions transform like I_3, i.e., like the third component of an isovector. Consider the reaction $\rho+N^*\to N+\pi$ where ρ denotes a spurion with $I=1$. There are two isospin invariants one can form, viz.,

$$f_1(\rho\times N^*)_{I=1/2}(N\times\pi)_{I=1/2}$$

and

$$f_3(\rho\times N^*)_{I=3/2}(N\times\pi)_{I=3/2}\,,$$

where f_1 and f_3 are coupling constants. We now put the $I_3=0$ component of ρ equal to unity and the other components equal to zero, in these invariants, to obtain $[N^*N\pi]$—Yukawa couplings which transform like I_3. Thus, the six coupling constants $F(N_3^*,p\pi^+)$, etc., for broken isospin symmetry are given in terms of three parameters f_0, f_1, and f_3. Consequently, for this first-

* On leave from the Tata Institute of Fundamental Research, Bombay, India.
† Present address: Tata Institute of Fundamental Research, Bombay, India.

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[6] M. Muraskin and S. L. Glashow, Phys. Rev. **132**, 482 (1963).
[7] C. Dullemond, A. J. Macfarlane, and E. C. G. Sudarshan, Phys. Rev. Letters **10**, 423 (1963) have already discussed this problem by Shumeskevich's method. However, these results do not agree with ours and are incorrect.

order breaking, we obtain the three sum rules

$$F(N_3^*,p\pi^+)+\sqrt{3}F(N_{-1}^*,p\pi^-)=6^{1/2}F(N_1^*,p\pi^\circ)\,,$$
$$\sqrt{3}F(N_1^*\to n\pi^+)+F(N_{-3}^*,n\pi^-)=6^{1/2}F(N_{-1}^*,n\pi^\circ)\,,$$
$$(\tfrac{3}{2})^{1/2}F(N_1^*,p\pi^\circ)+(\tfrac{3}{2})^{1/2}F(N_{-1}^*,n\pi^\circ)=F(N_3^*,p\pi^+)+F(N_{-3}^*,n\pi^-)\,.$$

It is clear this can be applied to any process $(I=\frac{3}{2})\to(I=\frac{1}{2})+(I=1)$. Other cases can be worked out in a similar manner and the sum rules so obtained may be checked experimentally for nuclei.

In the next section we generalize the spurion method for SU(3) and apply it to the $[DBP]$ coupling.

III. SUM RULES FOR THE [DBP] COUPLING CONSTANTS

1. Exact SU(3)

For exact SU(3) the Yukawa interaction

$$D\to B+P\,, \tag{1}$$

is characterized by only one coupling constant G_0. The twelve coupling constants $G(N^*,N\pi)$, etc., are, in terms of G_0, given by[8]

$$\begin{aligned}
&G(N^*,N\pi)=-G(N^*,\Sigma K)=-G_0/\sqrt{2}\,,\\
&G(Y_1^*,\Sigma\pi)=G(Y_1^*,\Xi K)=-G(Y_1^*,N\bar{K})=G_0/\sqrt{6}\,,\\
&G(Y_1^*,\Lambda\pi)=-G(Y_1^*,\Sigma\eta)=-G_0/2\,,\\
&G(\Xi^*,\Xi\pi)=G(\Xi^*,\Xi\eta)=G(\Xi^*,\Sigma\bar{K})=-G(\Xi^*,\Lambda\bar{K})=G_0/2\,,\\
&G(\Omega,\Xi\bar{K})=G_0.
\end{aligned} \tag{2}$$

2. Broken SU(3)

Following earlier work[1,4–7] we assume that the symmetry-breaking interaction transforms like the $I=0$, $Y=0$ component of an octet, i.e., like[1] λ_8. To obtain Yukawa couplings $[DBP]$ which transform like λ_8, we make use of a spurion octet S and consider the reaction

$$S+D\to B+P. \tag{3}$$

Let $D(I,Y)$ stand for the isospin equal to I and hypercharge equal to Y component of the decuplet. We use similar notation for the octets. Then, the state $\Psi^{(I,Y)}(I_1,Y_1;I_2,Y_2)$ with isospin I and hypercharge Y formed out of $B(I_1,Y_1)$ and $P(I_2,Y_2)$, is given by

$$\Psi^{(I,Y)}(I_1,Y_1;I_2,Y_2)=\textstyle\sum_\mu d_\mu^{(I,Y)}(I_1,Y_1;I_2,Y_2)\chi^{(\mu,I,Y)}\,, \tag{4}$$

where $\chi^{(\mu,I,Y)}$ is the state vector with isospin I and hypercharge Y in the representation of SU(3) characterized by μ. The sum over the representations in (4) is for $\mu=1$, 8_1, 8_2, 10, 10^*, 27. Similarly, for the product of the spurion component $S(I_1,Y_1)$ and $D(I_2,Y_2)$ we can define the state $\Phi^{(I,Y)}(I_1,Y_1;I_2,Y_2)$. However, as only the $I=0$, $Y=0$ component of the spurion is nonvanishing, we are only interested in the case $I_1=0$, $Y_1=0$. Thus, we write

$$\Phi^{(I,Y)}(0,0;I,Y)=\textstyle\sum_\nu C_\nu^{(I,Y)}\chi^{(\nu,I,Y)}\,, \tag{5}$$

where the sum over the representations ν is for $\nu=8$, 10, 27, 35. It is clear from (4) and (5) that there will be four Yukawa couplings which transform as λ_8 with the coupling constants $G_{(\mu,\nu)}$, with $(\mu,\nu)=(27,27)$, (10,10), $(8_1,8)$, and $(8_2,8)$. Thus, the total effective coupling constants $G[D(I,Y),B(I_1,Y_1)P(I_2,Y_2)]$, where $\mathbf{I}=\mathbf{I}_1+\mathbf{I}_2$ and $Y=Y_1+Y_2$, with first order breaking are given by

$$G[D(I,Y),B(I_1,Y_1)P(I_2,Y_2)]=d_{10}^{(I,Y)}(I_1,Y_1;I_2,Y_2)G_0+\textstyle\sum_{(\mu\nu)}C_\mu^{(I,Y)}d_\nu^{(I,Y)}(I_1,Y_1;I_2,Y_2)G_{(\mu,\nu)}. \tag{6}$$

The sum in (6) has four terms for $(\mu,\nu)=(27,27)$, (10,10), $(8_1,8)$ and $(8_2,8)$. There are twelve G's expressed by (6) in terms of the five parameters G_0, $G_{27,27}$, $G_{10,10}$, $G_{8_1,8}$, and $G_{8_2,8}$. Consequently, we can eliminate the five parameters to obtain seven sum rules. Using (6) and the values of the coefficients[8] C's and d's, we obtain

$$X(N^*,N\pi)=-\sqrt{2}G(N^*,N\pi)=a+(5/4)p-s\,, \tag{7.1}$$
$$X(N^*,\Sigma K)=\sqrt{2}G(N^*,\Sigma K)=a-(5/4)p-s\,, \tag{7.2}$$
$$X(Y_1^*,N\bar{K})=-6^{1/2}G(Y_1^*,N\bar{K})=a+p-3q+r\,, \tag{7.3}$$
$$X(Y_1^*,\Lambda\pi)=-2G(Y_1^*,\Lambda\pi)=a+p+2q\,, \tag{7.4}$$
$$X(Y_1^*,\Sigma\pi)=6^{1/2}G(Y_1^*,\Sigma\pi)=a-2r\,, \tag{7.5}$$
$$X(Y_1^*,\Sigma\eta)=2G(Y_1^*,\Sigma\eta)=a-p-2q\,, \tag{7.6}$$
$$X(Y_1^*,\Xi K)=6^{1/2}G(Y_1^*,\Xi K)=a-p+3q+r\,, \tag{7.7}$$
$$X(\Xi^*,\Lambda\bar{K})=-2G(\Xi^*,\Lambda\bar{K})=a+\tfrac{3}{4}p-q+r+s\,, \tag{7.8}$$
$$X(\Xi^*,\Sigma\bar{K})=2G(\Xi^*,\Sigma\bar{K})=a-\tfrac{1}{4}p-3q-r+s\,, \tag{7.9}$$
$$X(\Xi^*,\Xi\eta)=2G(\Xi^*,\Xi\eta)=a-\tfrac{3}{4}p+q+r+s\,, \tag{7.10}$$
$$X(\Xi^*,\Xi\pi)=2G(\Xi^*,\Xi\pi)=a+\tfrac{1}{4}p+3q-r+s\,, \tag{7.11}$$
$$X(\Omega,\Xi\bar{K})=G(\Omega,\Xi\bar{K})=a+2s\,, \tag{7.12}$$

where we have put

$$a=G_0,\quad p=\tfrac{3}{5}G_{(27,27)},\quad q=\tfrac{1}{5}G_{(8_1,8)},\quad r=(\tfrac{1}{5})^{1/2}G_{(8_2,8)}\,,$$

and

$$s=\tfrac{1}{4}\sqrt{2}G_{(10,10)}. \tag{8}$$

Eliminating the parameters, a, p, etc., we obtain the sum rules

$$\tfrac{1}{2}[X(N^*,N\pi)+X(\Xi^*,\Xi\pi)]=\tfrac{1}{4}[3X(Y_1^*,\Lambda\pi)+X(Y_1^*,\Sigma\pi)]\,, \tag{9.1}$$
$$\tfrac{1}{2}[X(N^*,\Sigma K)+X(\Xi^*,\Sigma\bar{K})]=\tfrac{1}{4}[3X(Y_1^*,\Sigma\eta)+X(Y_1^*,\Sigma\pi)]\,, \tag{9.2}$$
$$\tfrac{1}{2}[X(Y_1^*,N\bar{K})+X(\Omega,\Xi\bar{K})]=\tfrac{1}{4}[3X(\Xi^*,\Lambda\bar{K})+X(\Xi^*,\Sigma\bar{K})]\,, \tag{9.3}$$

[8] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963). The necessary Clebsch-Gordan coefficients for SU(3) are listed here.

$$\tfrac{1}{2}[X(Y_1^*,\Xi K)+X(\Omega,\Xi\bar{K})] = \tfrac{1}{4}[3X(\Xi^*,\Xi\eta)+X(\Xi^*,\Xi\pi)], \quad (9.4)$$

$$X(Y_1^*,\Sigma\pi)+X(\Omega,\Xi\bar{K})=X(\Xi^*,\Xi\pi)+X(\Xi^*,\Sigma\bar{K}), \quad (9.5)$$

$$X(N^*,N\pi)+X(\Xi^*,\Sigma\bar{K}) = X(Y_1^*,N\bar{K})+X(Y_1^*,\Sigma\pi), \quad (9.6)$$

$$X(N^*,\Sigma K)+X(\Xi^*,\Xi\pi) = X(Y_1^*,\Xi K)+X(Y_1^*,\Sigma\pi). \quad (9.7)$$

The sum rules (9.1)–(9.4) have the same structure as the Gell-Mann–Okubo mass formula for the unitary octets.

3. Sum Rules for $10^* \to 8+8$

In case there exists a resonance multiplet corresponding to the 10^* representation of SU(3) which decays into two octets then one obtains the sum rules, for this case, as indicated below.

Let the particles of the 10^* multiplet be denoted by Ξ', Y_1', N' and Ω' with $(I=\frac{3}{2}, Y=-1)$, $(I=1, Y=0)$, $(I=\frac{1}{2}, Y=1)$, and $(I=0, Y=+2)$, respectively. Then for exact SU(3), we have[8]

$$G(\Xi',\Xi\pi)=-G(\Xi',\Sigma\bar{K})=-F_0/\sqrt{2}\,,$$
$$G(Y_1',\Sigma\pi)=-G(Y_1',N\bar{K})=G(Y_1',\Xi K)=-F_0/\sqrt{6}\,,$$
$$G(Y_1',\Lambda\pi)=-G(Y_1',\Sigma\eta)=-F_0/2\,,$$
$$G(N',N\pi)=-G(N',N\eta)=G(N',\Sigma K) =+G(N',\Lambda K)=-F_0/2\,,$$
$$G(\Omega',NK)=-F_0.$$

Corresponding to the X's for $D\to B+P$, we define, $Z(\Xi',\Xi\pi)$, $\cdots$, $Z(\Omega',NK)$ to be the ratios of the corresponding coupling constants for broken SU(3) case to those for the exact SU(3) limit for the couplings $10^*\to 8+8$. Then, the sum rules in terms of the Z's can be obtained from the Eqs. (9), by the following replacements (i.e., essentially R conjugation):

$$N^*\to\Xi',\ Y_1^*\to Y_1',\ \Xi^*\to N',\ \Omega\to\Omega'$$
$$N\to\Xi,\ \Sigma\to\Sigma,\ \Lambda\to\Lambda,\ \Xi\to N,$$

and

$$K\to\bar{K},\ \bar{K}\to K,\ \eta\to\eta,\ \pi\to\pi.$$

IV. DISCUSSION OF THE SUM RULES

The sum rule (9.1) clearly involves coupling constants which can be determined from experiment as mentioned earlier. Since the four decays are p wave, the width is given by

$$\Gamma=|G|^2p^3m_B/m_D\,, \quad (10)$$

where m_D is the mass of the resonance which decays into the baryon of mass m_B and p the final c.m. momentum. Using (10) and the data given recently by Roos[9] we give the values of the G's in Table I. The sum rule (9.1) in terms of the G's is

TABLE I. The coupling constants $G(N^*,N\pi)$, etc. calculated from the experimental widths (Ref. 9). The relative signs of the coupling constants are taken in accordance with the exact SU(3) limit. We have taken the branching ratio for $Y_1^*\to\Sigma\pi$ to be 1%.

m_D(MeV)	Decay	Exptl. width (MeV)	G(BeV)$^{-1}$
1238	$N^*\to N\pi$	94.0 ±16.0	3.127±0.266
1385	$Y_1^*\to\Lambda\pi$	38.61± 6.93	2.269±0.204
1385	$Y_1^*\to\Sigma\pi$	0.39± 0.07	−0.520±0.046
1530	$\Xi^*\to\Xi\pi$	7.0 ± 2.0	−1.575±0.225

$$\sqrt{2}G(N^*,N\pi)-2G(\Xi^*,\Xi\pi) =3G(Y_1^*,\Lambda\pi)-(\tfrac{3}{2})^{1/2}G(Y_1^*,\Sigma\pi). \quad (11)$$

Using the values in Table I, the left-hand side of $(11)=7.583\pm0.826$ (BeV)$^{-1}$ and the right-hand side of $(11)=7.444\pm0.668$ (BeV)$^{-1}$. Thus, the sum rule is rather well satisfied experimentally.

There is little or no information available for the other coupling constants. However, an analysis of $\bar{K}N$-data could give an estimate of $G(Y_1^*,\bar{K}N)$. Then, one could estimate

$$G(\Omega,\Xi\bar{K})=-6^{1/2}G(Y_1^*,N\bar{K}) +2G(\Xi^*,\Xi\pi)+\sqrt{2}G(N^*,N\pi)\,, \quad (12.1)$$

$$2G(\Xi^*,\Sigma\bar{K})=-6^{1/2}G(Y_1^*,N\bar{K}) +6^{1/2}G(Y_1^*,\Sigma\pi)+\sqrt{2}G(N^*,N\pi)\,, \quad (12.2)$$

$$2G(\Xi^*,\Lambda\bar{K})=6^{1/2}G(Y_1^*,N\bar{K}) -\sqrt{2}G(N^*,N\pi)+2G(Y_1^*,\Lambda\pi)\,, \quad (12.3)$$

in terms of $G(Y_1^*,N\bar{K})$ and known quantities.[10–12]

Finally, we may express the rest of the coupling constants in terms of three known ones, viz., $G(N^*,N\pi)$, $G(Y_1^*,\Lambda\pi)$, $G(\Xi^*,\Xi\pi)$, and $G(Y_1^*,N\bar{K})$, $G(Y_1^*,\Xi K)$ as follows:

$$\sqrt{2}G(N^*,\Sigma K)=6^{1/2}G(Y_1^*,\Xi K)-2\sqrt{2}G(N^*,N\pi) +2G(\Xi^*,\Xi\pi)+6G(Y_1^*,\Lambda\pi)\,, \quad (13.1)$$

$$2G(Y_1^*,\Sigma\eta)=[2(6)^{1/2}/3][G(Y_1^*,\Xi K)-G(Y_1^*,N\bar{K})] +(\tfrac{4}{3})[2G(\Xi^*,\Xi\pi)-\sqrt{2}G(N^*,N\pi)] +6G(Y_1^*,\Lambda\pi)\,, \quad (13.2)$$

$$2G(\Xi^*,\Xi\eta)=[2(6)^{1/2}/3] \times[G(Y_1^*,\Xi K)-G(Y_1^*,N\bar{K})] +\tfrac{1}{3}[2G(\Xi^*,\Xi\pi)+2\sqrt{2}G(N^*,N\pi)]. \quad (13.3)$$

In conclusion, it may be emphasized that all the above sum rules are obtained by taking into account the symmetry breaking only up to the first order. Also, they obviously apply to any decuplet-octet-octet coupling constants.

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